

have been to add the form akin to “on 1st of May.” It gives the score [1.2, 2.2; 0.6, 16.4].

The eight regular date forms in Table 1 can be used in $2^8 - 1 = 255$ non-empty combinations, of which WRR used one combination (i.e., the first three). We tried all 255 combinations and found that WRR’s choice was uniquely the best for the first and fourth of our four success measures. In the case of our second measure (least permutation rank of P_{1-4} for the first list), WRR’s choice is sixth best. (The best is a subset of their three forms.) For our third measure (P_4 for the second list), WRR’s choice is third best. Since the various date forms are not equal in their frequency of use, it would be unwise to form a quantitative conclusion from these observations.

APPENDIX C: VARIATIONS OF THE METRIC

This Appendix gives the technical details for the variations we tried on WRR’s method of analysis. In all cases presented here, the text of Genesis and the list of word pairs was held fixed. A deep understanding of the metric is needed for this Appendix, for which we refer the reader to Appendix A.

First consider the function $\delta_h(e, e')$ that lies at the heart of the WRR metric. Define these quantities:

$$f = \Delta(d, h),$$

$$f' = \Delta(d', h),$$

$$l = \min \Delta(|n + di - n' - d'i'|, h),$$

$$\mu = \text{mean} \Delta(|n + di - n' - d'i'|, h),$$

$$m = \Delta(|2n + d(k-1) - 2n' - d'(k'-1)|/2, h),$$

$$L = \max \Delta(|n + di - n' - d'i'|, h),$$

x, y = dimensions of smallest enclosing rectangle,

where the min, mean and max are taken over $0 \leq i \leq k-1$ and $0 \leq i' \leq k'-1$. The quantity m is the cylindrical distance between the midpoints of the two ELSs.

WRR define $\delta_h(e, e') = f^2 + f'^2 + l^2$, which is a square of a distance. In Table 5 we show the effects of making other choices. We have restricted ourselves to distances and squares of distances, and to functions which measure the same type of compactness that WRR’s function measures. The latter condition is enforced in a strong sense: for bounded word length, each function in Table 5 is bounded above and below by moderate constant multiples of the first. For example, $f^2 + f'^2 + l^2 \leq (f + f' + l)^2 \leq 3(f^2 + f'^2 + l^2)$.

The paucity of values less than 1 in the table and their blandness is remarkable. We did not find a sin-

TABLE 5
The effect of changing $\delta_h(e, e')$

$\phi(e, e')$	$\delta(e, e') = \phi(e, e')$	$\delta(e, e') = \sqrt{\phi(e, e')}$
$f^2 + f'^2 + l^2$	[1, 1; 1, 1] (WRR)	[154, 120; 10.1, 99]
$f^2 + f'^2 + m^2$	[1.5, 3.7; 66, 92]	[65, 83; 101, 650]
$f^2 + f'^2 + \mu^2$	[1.3, 5.1; <u>0.6</u> , 2.3]	[168, 230; 25, 410]
$f^2 + f'^2 + L^2$	[2.4, 4.1; 1.0, 11.4]	[220, 340; 40, 1000]
$f^2 + f'^2 + 2l^2$	[2.5, 1.6; 2.8, 1.1]	[210, 88; 12.1, 66]
$2f^2 + 2f'^2 + l^2$	[1.4, 1.3; <u>0.6</u> , 1.8]	[61, 82; 11.7, 220]
$(f + f' + l)^2$	[1.8, 1.9; <u>0.5</u> , 1.0]	[190, 137; 10.1, 154]
$(f + f' + m)^2$	[<u>0.6</u> , 1.9; 17.5, 57]	[98, 120; 130, 1200]
$(f + f' + \mu)^2$	[3.6, 8.3; <u>0.4</u> , 3.7]	[220, 290; 20, 550]
$(f + f' + L)^2$	[7.1, 15.1; <u>0.5</u> , 11.6]	[430, 460; 34, 1100]
$\max(f, f', l)^2$	[2.4, 1.3; 2.7, 1.9]	[86, 76; 6.8, 69]
$\max(f, f', m)^2$	[3.9, 6.8; 240, 230]	[40, 58; 74, 400]
$\max(f, f', \mu)^2$	[2.9, 9.8; 1.2, 3.0]	[220, 280; 25, 310]
$\max(f, f', L)^2$	[2.5, 13.3; 1.1, 12.1]	[380, 500; 39, 810]
μ^2	[5.7, 18.6; 2.2, 4.2]	[340, 360; 49, 420]
L^2	[2.8, 13.6; 1.3, 12.3]	[420, 530; 35, 740]
$(L + l)^2$	[4.0, 13.8; 2.1, 7.0]	[360, 380; 73, 570]
$L^2 + l^2$	[2.7, 13.4; <u>0.9</u> , 5.5]	[330, 450; 38, 600]
$(x + y)^2$	[30, 44; <u>0.5</u> , 16.8]	[640, 550; 15.5, 630]
$x^2 + y^2$	[15.1, 33; <u>0.4</u> , 9.7]	[500, 610; 18.5, 620]
$\max(x, y)^2$	[9.9, 31; <u>0.2</u> , 5.9]	[190, 340; 31, 840]
xy	[680, 140; <u>0.5</u> , 71]	[1.1e4, 720; 97, 3900]
$x^2 + y^2 + l^2$	[8.9, 26; <u>0.4</u> , 4.7]	[180, 320; 24, 740]
$x^2 + y^2 + m^2$	[1.5, 13.2; 2.3, 14.4]	[150, 340; 26, 830]
$x^2 + y^2 + \mu^2$	[7.4, 24; <u>0.5</u> , 5.4]	[183, 310; 23, 680]
$x^2 + y^2 + L^2$	[14.7, 38; <u>0.7</u> , 8.2]	[430, 560; 27, 720]
$(x + y + l)^2$	[7.1, 17.4; <u>0.1</u> , 1.1]	[250, 290; 21, 440]
$(x + y + m)^2$	[2.0, 13.7; 1.9, 13.5]	[230, 380; 28, 705]
$(x + y + \mu)^2$	[22, 22; <u>0.3</u> , 4.3]	[430, 500; 22, 650]
$(x + y + L)^2$	[10.4, 26; <u>0.8</u> , 12.6]	[610, 630; 37, 1100]
$xy + l^2$	[42, 28; <u>0.3</u> , 1.4]	[3900, 600; 46, 211]
$xy + m^2$	[4.0, 17.3; 3.8, 26]	[670, 440; 74, 830]
$xy + \mu^2$	[11.6, 27; <u>0.4</u> , 3.2]	[740, 560; 49, 650]
$xy + L^2$	[9.4, 26; <u>0.9</u> , 15.0]	[810, 710; 43, 1050]

gle variation that improved the result of the permutation test for either list. In the case of the first list, only one variation improved P_2 , and then only by a little. Only the P_4 value for the second list shows a significant number of improvements (19 out of 67), which is not too surprising in light of the fact that P_4 was not the only criterion of success. In this regard, we mention that only 6 of the 67 variations in the table increase the value of P_4 for the distances after the cyclic shift of the dates (another of WRR’s success measures, but one they wanted to be large; see Sections 3 and 7). Similarly, only 4 of the 67 variations improve the flatness of the histogram of those distances, as measured by the χ^2 statistic with 25 bins (the same bins displayed in WRR94).

Furthermore, in all 19 cases where P_4 dropped, the permutation rank of P_4 increased. This indicates that the observed drop in P_4 values is due to an overall tendency for $c(w, w')$ values to decrease

when these variations are applied. In other words, it is an example of the inadequacy of P_4 as an indirect indicator of tuning, as discussed in Section 7.

The second step is the computation of $\mu_h(e, e')$ from $\delta_h(e, e')$. The mapping must have negative derivative, but WRR's choice $\mu = 1/\delta$ is not the only possibility. Other possibilities are included in Table 6 (though the first is already in Table 5). Table 6 also shows the effect of slight changes to the definition of $H(d, d')$.

The practice of using the perturbed letter positions for measuring distances, introduced by WRR some time after the completion of the work reported in WRR94, has only a slight effect for both lists: [0.8, 0.7; 1.2, 0.9]. Their other major change, replacing the definition of $\Delta(n, h)$ by one that is more geometrically correct, has a negligible effect.

The value $\sigma(e, e')$ is defined as a sum over h , but, as mentioned by WRR (1986), it could have been the maximum instead. That gives [176, 6.3; 12.6, 3.9]. If we are looking for the best term, we could also widen the search by including the values of h on each side of those in $H(d, d')$ [280, 7.9; 26, 17], or two values on each side [420, 11.2; 21, 15].

The definition of domain of minimality allows variation too. Instead of “smaller than d ,” we could use “smaller than or equal to d ,” or just take the whole text. Similarly, instead of using the size of

TABLE 6
The effect of changing $\mu_h(e, e')$ or $H(d, d')$

Variation	Scores
<i>Definition of $\mu_h(e, e')$</i>	
$1/\sqrt{\delta}$	[154, 120, 10.1, 99]
$1/\delta^2$	[560, 6.0, 26, 2.5]
$-\delta$	[5e8, 6100, 1e8, 7e5]
$-\delta^2$	[5e8, 2e4, 1e8, 7e5]
$-\ln \delta$	[6e8, 3000, 1e8, 8e5]
$\exp(-\delta)$	[3e6, 240, 250, 33]
<i>Definition of $H(d, d')$</i>	
Round $\frac{1}{2}$ down	[1.1, 1.0; 1.4, 1.5]
Always round down	[0.8, 0.8; 1.5, 1.6]
Always round up	[1.4, 1.0; 0.4, 0.6]
Remove duplicates	[0.5, 0.7; 1.5, 1.7]
Use 1 value of i	[2e5, 340; 31, 21]
or 2	[2e4, 210; 3.4, 4.5]
or 5	[3.7, 0.6; 0.3, 0.2]
or 10 (WRR)	[1, 1; 1, 1]
or 15	[3.6, 3.3; 1.4, 1.1]
or 20	[11.8, 5.9; 3.1, 3.8]
or 25	[66, 15.3; 4.8, 5.4]
or 50	[3600, 40; 93, 28]
Minimum row length 3	[0.9, 1.0; 1.3, 1.2]
or 4	[0.9, 1.0; 1.0, 1.1]
or 5	[0.9, 1.0; 1.2, 1.3]
or 10	[1.1, 0.9; 5.4, 5.9]

TABLE 7
Various definitions of domains of minimality

Variation	Scores
<i>Definition of T_e</i>	
Use $\leq$	[1.3, 1.1; 3.7, 2.7]
Whole text	[27, 850; 2.0, 407]
<i>Definition of $L\omega(e, e')$</i>	
$ T_e \cap T_{e'} ^2$	[36, 1.5; 12.1, 1.1]
$ T_e \cup T_{e'} $	[94, 580; 0.2, 29.1]
... but only if disjoint	[27, 52; 0.5, 19.0]
$ T_e T_{e'} $	[4.6, 1.3; 2.2, 0.8]
$(T_e + T_{e'})/2$	[4.8, 42, 0.5, 11.9]
$\sqrt{ T_e T_{e'} }$	[2.7, 5.8; 0.8, 6.3]
$\min(T_e , T_{e'})$	[1.1, 1.7; 0.9, 1.1]
$\max(T_e , T_{e'})$	[109, 470; 0.4, 27]

the intersection to define the domain of simultaneous minimality, we could use the square of the intersection or other functions. Table 7 gives the scores.

Next consider the definition of the key function $\Omega(w, w')$. WRR defined it as a sum, but they could also have taken the best term [4700, 13.6; 64, 1.8]. If the best term is taken there, it makes sense to also take the best term in defining σ [2e5, 12.5; 690, 10.2], perhaps with the search expanded to more h values, as described above: [1e5, 23; 2200, 52] and [9e4, 22; 2900, 100].

Another important part of the definition of $\Omega(w, w')$ is the definition of $E(w)$. WRR define it according to a skip limit with parameter 10 (an expected number of ELSs, as described before). The value 10 is not sacred; in fact, it is stated in WRR94 that a limit was only used to reduce the computational effort. However, as Table 8 shows, there is a

TABLE 8
The effect of changing $E(w)$

Variation	Scores
<i>Expected ELS count of 2</i>	
or 5	[7600, 7.0; 4e4, 310]
or 10 (WRR)	[53, 1.6; 20, 19.5]
or 15	[1, 1; 1, 1]
or 20	[1.2, 2.9; 5.9, 2.0]
or 25	[2.7, 8.3; 59, 7.1]
or 30	[0.8, 4.0; 91, 15.2]
or 50	[6.8, 14.1; 144, 22]
or 75	[2.2, 4.1; 550, 79]
or 100	[3.7, 4.5; 590, 81]
Exactly 10 ELSs	[4.0, 4.7; 560, 62]
Minimum skip of 1	[23, 2.2; 630, 7.7]
or 3	[1.5, 2.1; 0.1, 5.0]
or 4	[0.3, 0.7; 11.1, 5.9]
or 5	[1.2, 1.6; 16.3, 7.9]
or 10	[0.5, 0.8; 16.7, 11.3]
	[13.7, 0.6; 33, 35]

clear optimum near 10 for both lists! (As an aside, we note that if we take WRR at their word that the bound of 10 was only for computational efficiency, we must conclude that the “true” result of their experiment was one or two orders of magnitude weaker than claimed.)

The sharp cut-off at parameter 10 allows us a simple experiment which to some extent is independent of the original experiment. We did the same computation restricted to those ELS pairs which lie within the cut-off at parameter 20 but not within the cut-off at parameter 10. Out of all eight statistics (P_{1-4} for each list), there is no value less than 0.418 and no permutation rank less than 0.342.

The use of the correct formula for defining $E(w)$ (see Appendix A), or whether the boundary is rounded up or down, have no effect (to the accuracy we are measuring it). However, some other variations do have an effect. Choosing the 10 ELSs with least skip, rather than all those within a boundary chosen to give 10 on average, affects the result a lot, as does using a lower bound other than 2 for the skip. These results, shown in Table 8, show that the result for the second list owes a lot to ELSs with very small skips, at which scales the strong nonrandomness of the text makes the method of perturbations nonsensical.

Next we consider the definition of the perturbations (x, y, z) . Instead of applying them to the last three letters, we could follow the diagram given originally by WRR (1986) (but apparently not used in the calculations there) and apply them always to the third, fourth and fifth letters, or we could apply them in pattern x, y, z instead of $x, x+y, x+y+z$. We could also try perturbing two letters instead of three, or perturbing them by larger amounts. Another variation in the use of perturbations, suggested by Witztum, is to only perturb the ELSs for the dates and use unperturbed ELSs for the appellations. We tried it the other way round as well. The scores for all these variations appear in Table 9.

TABLE 9
Different ways to do perturbations

Variation	Scores
Perturb as x, y, z	[0.7, 0.1; 0.8, 2.1]
Perturb letters 3, 4, 5	[0.4, 1.0; 1.3, 2.1]
Perturb up to 3 places	[0.2, 2.4; 0.04, 1.1]
or 4 places	[0.2, 4.2; 0.005, 0.6]
Perturb last 2 letters	[5e4, 4.5; 6700, 28]
up to 3 places	[118, 2.4; 340, 18.6]
or 4 places	[2.5, 0.6; 135, 48]
Perturb only appellations	[23, 7.5; 240, 34]
Perturb only dates	[15000, 0.3; 1350, 7.3]

TABLE 10
Different denominator bounds or P_1 cutoffs

Variation	Scores
<i>Denominator bound</i>	
2	[2.9, 1.0; 1.0, 1.0]
3	[2.9, 1.2; 1.0, 1.0]
4	[1.8, 1.2; 1.0, 1.0]
5	[1.8, 1.2; 1.0, 1.0]
15	[1.0, 1.0; 1.0, 1.0]
20	[1.0, 0.9; 1.1, 1.1]
25	[1.0, 1.0; 1.1, 1.1]
<i>Cutoff defining P_1</i>	
0.05	[1, 1.0; 1, 1.0]
0.1	[1, 1.0; 1, 1.0]
0.15	[1, 1.0; 1, 1.0]
0.25	[1, 0.8; 1, 1.0]
0.33	[1, 1.0; 1, 1.0]
0.4	[1, 1.0; 1, 1.0]
0.50	[1, 0.4; 1, 1.0]

Note that using perturbation amounts as large as the skip is absurd, as different letters can be sought at the same position in the text. The two very small P_4 ratios (0.04 and 0.005) in the table are artifacts caused by that anomaly. Restricting the skip to be greater than the maximum perturbation increases them to 1.9 and 0.2, respectively.

Table 10 shows the effects of the lower bound 10 for the number of defined $\Omega^{(x, y, z)}(w, w')$ values, appearing in the definition of $c(w, w')$. The same table shows the effect of changing the cut-off 0.2 used to compute P_1 and P_3 . Values greater than 0.2 have a dramatic effect on P_1 , reducing it by a large factor (especially for the first list). However, the result of the permutation test on P_1 does not improve so much, and for the second list it is never better than that for P_4 .

In applying the permutation test, there are a few more possible variations. Some rabbis have either no dates or no appellations in WRR's lists. In one case, they selected no appellations within their self-imposed length bounds of 5–8 letters. In other cases, they eliminated dates on the grounds that they were uncertain. Removing such rabbis has a minor effect, [1.0, 1.2; 1.0, 0.9]. In addition, some of the other rabbis produce no distances either (because of appellations or dates having no ELSs); removing all rabbis that produce no distances has the effect [1.0, 0.4; 1.0, 7.8].

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